



ANALYSIS OF THE STIRLING HEAT ENGINE AT MAXIMUM POWER CONDITIONS

L. BERRIN ERBAY[†] and HASBI YAVUZ[‡]

[†]Osmangazi University, School of Engineering, 26030, Eskişehir, Turkey and [‡]Istanbul Technical University, Institute for Nuclear Energy, 80626, Istanbul, Turkey

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Abstract—The Stirling heat engine operating in a closed regenerative thermodynamic cycle is analyzed. Polytropic processes are used for the power and displacement pistons. Following regeneration, the maximum power density and efficiency are found and the compression ratio at maximum power density is determined. © 1997 Elsevier Science Ltd.

INTRODUCTION

The use of a regenerative heat exchanger (RHX) distinguishes the Stirling heat engine from the Carnot engine [1–7]. A power analysis of an endoreversible Stirling cycle was conducted previously and the maximum power (MP) and efficiency at maximum power ($\eta_{w_{max}}$) based on higher and lower temperature bounds were obtained for the cycle with perfect regeneration [8]. The primary purpose of this study is to obtain a more realistic regenerator, which is obtained by introducing polytropic processes at the power and displacement pistons. By application of the MPD technique [9], maximum power density (MPD) conditions of the Stirling heat engine with realistic RHX are investigated.

GENERAL THEORY AND MODEL

A schematic of the equipment and the T - s diagram of the Stirling cycle for an ideal-gas working fluid are shown in Figs. 1 and 2, respectively. The exponents of the polytropic processes must be within the limits $0 < n_t < 1$ and $0 < n_c < 1$. The cycle compression and temperature ratio parameters are introduced as $r_v = v_4/v_2 > 1$ and $\psi = T_2/T_4 < 1$, respectively. Using r_v and ψ , the ratios of the temperatures at the ends of each cycle branch become

$$T_2/T_1 = r_v^{n_c-1}, T_3/T_4 = r_v^{n_t-1}, T_1/T_4 = r_v^{1-n_c}, T_3/T_2 = r_v^{n_t-1}. \quad (1)$$

From Eq. (1), the net power of the Stirling cycle is

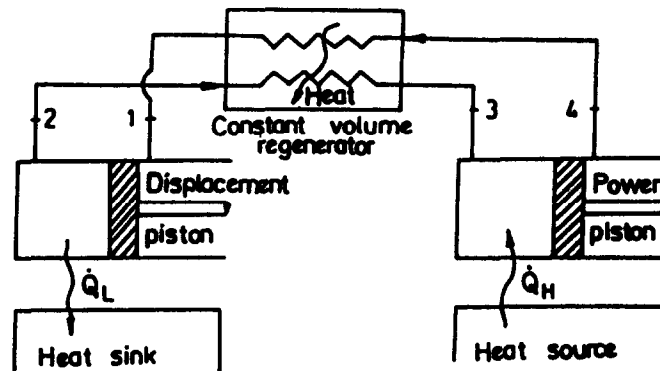
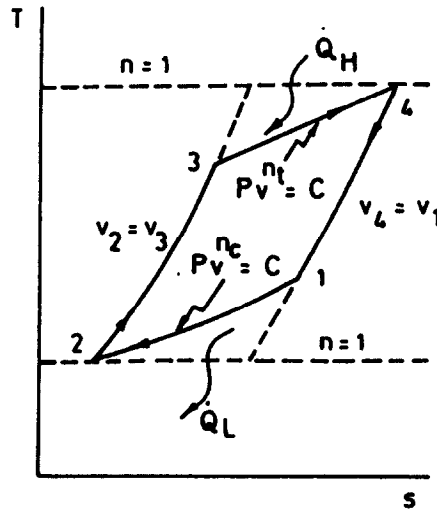


Fig. 1. Schematic of equipment for the Stirling cycle.

[‡]Author for correspondence. Fax: 90-212-285 3884, e-mail: NEYAVUZ@CC.ITU.EDU.TR

Fig. 2. T - s diagram for the Stirling cycle.

$$\dot{W}_{\text{net}} = \dot{m}RT_4 [n_t(1 - r_{vt}^{n_t})/(1 - n_t) - \psi n_c(r_v^{1-n_c} - 1)/(1 - n_c)]. \quad (2)$$

After equating heats transferred at the regenerator, the regeneration condition

$$\psi = (1 - r_{vt}^{n_t})/(r_v^{1-n_c} - 1) \quad (3)$$

is obtained. From Eq. (3),

$$n_c(\psi, n_t, r_v) = 1 - \ln[1 + (1/\psi)(1 - r_{vt}^{n_t})]/\ln r_v. \quad (4)$$

Then,

$$\dot{W}_{\text{net}} = \dot{m}RT_4(1 - r_{vt}^{n_t})\Phi, \quad (5)$$

where

$$\Phi = \left\{ n_t/(1 - n_t) - n_c(\psi, n_t, r_v)/[1 - n_c(\psi, n_t, r_v)] \right\}. \quad (6)$$

It is found from Eq. (5) that $n_t > n_c$. From Eq. (3), $r_v < \psi^{(n_t-1)^{-1}}$ is the requirement to ensure $n_t > n_c$. $\dot{W}_d$ is obtained by dividing Eq. (5) by v_4 , viz.,

$$\dot{W}_d = \dot{W}_{\text{net}}/v_4 = \dot{m}RT_4(r_v^{-1} - r_{vt}^{n_t-2})\Phi/v_2. \quad (7)$$

The transferred heat is

$$\dot{Q}_H = \dot{Q}_{34} = \dot{m} c_v T_4(1 - r_{vt}^{n_t})(k - n_t)/(1 - n_t). \quad (8)$$

In view of Eq. (3), the thermal efficiency of the cycle obtained from Eqs. (5) and (8) is

$$\eta = (k - 1)(1 - n_t)/(k - n_t)\Phi. \quad (9)$$

RESULTS

Since $\dot{W}_{\text{max}}$ and $\dot{W}_{d_{\text{max}}}$ are known, we may compare $(\dot{W}_{\text{net}}/\dot{W}_{\text{max}})$ and $(\dot{W}_{\text{max}}/\dot{W}_{d_{\text{max}}})$ as functions of η . As may be seen in Fig. 3, the efficiency at the MPD ($\eta_{\dot{W}_{d_{\text{max}}}}$) is greater than $\eta_{\dot{W}_{\text{max}}}$.

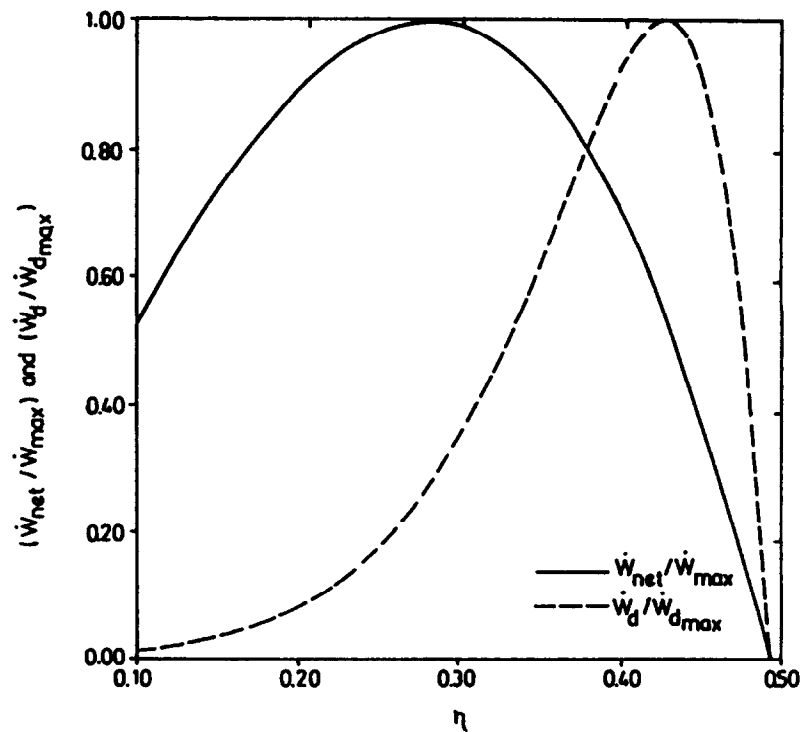


Fig. 3. Comparison of work ratios vs. η ($\psi = 0.2$, $n_t = 0.75$).

Figure 4 shows the effect of temperature on the two efficiencies $\eta_{\dot{W}_{\text{max}}}$ and $\eta_{\dot{W}_{d\text{max}}}$, where we have made use of ψ . It is seen in Fig. 4 that the $\eta_{\dot{W}_{d\text{max}}}$ values are higher than the $\eta_{\dot{W}_{\text{max}}}$ values at the same ψ . Figure 5 shows that the r_v 's at the MP are always greater than those of the MPD. Stirling heat engines designed by considering the MP conditions will have larger dimensions than those designed at the MPD conditions. The difference between the sizes of Stirling engines designed by using these

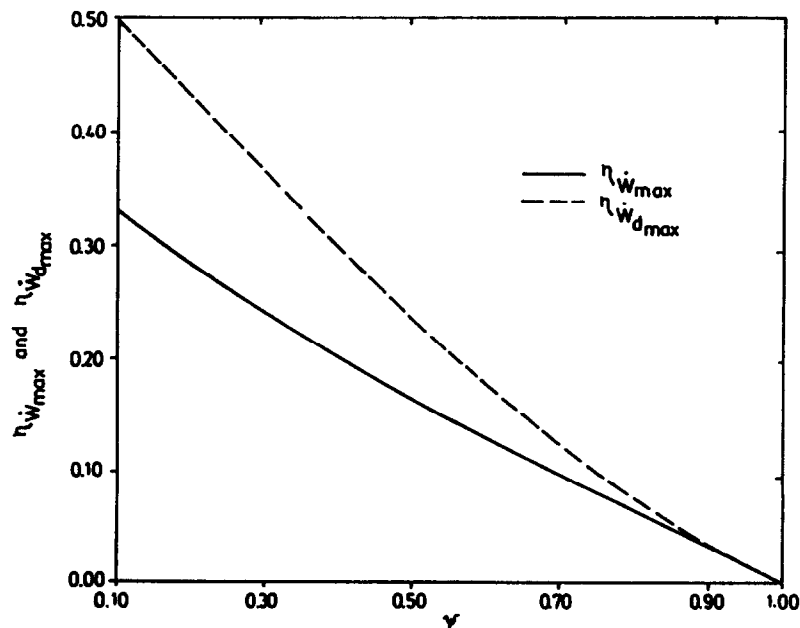


Fig. 4. Effects of cycle-temperature ratio ψ on the thermal efficiencies at maximum power and maximum power density ($n_t = 0.75$, $k = 1.4$).

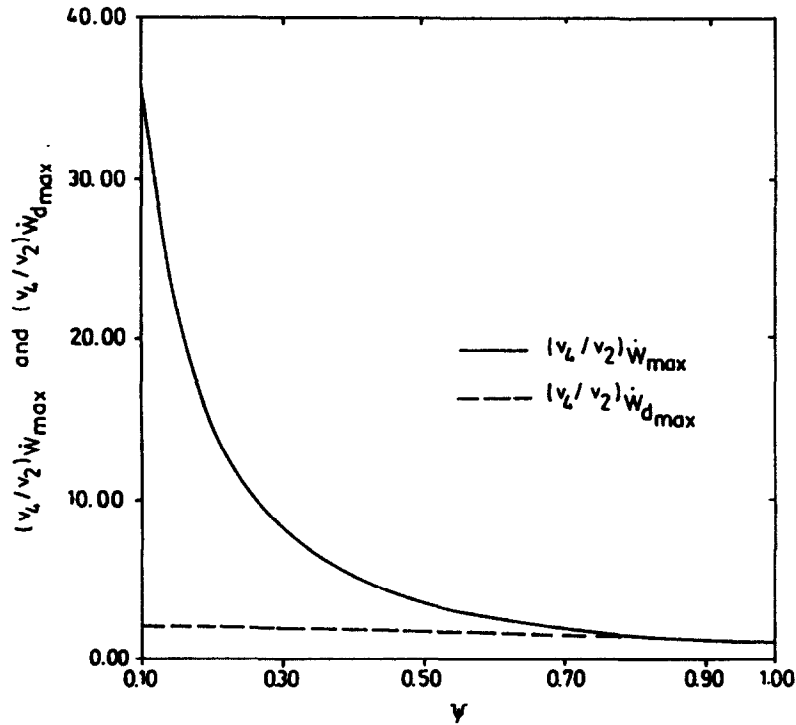


Fig. 5. Variations of the cycle compression ratios at maximum power $(v_4/v_2)\dot{w}_{\max}$ and at maximum power density $(v_4/v_2)\dot{w}_{d\max}$ with ψ ($n_t = 0.75$).

criteria is striking, especially for small ψ values. It is observed from Fig. 6 that η increases with decreasing r_c and increasing n_t .

The optimum working region of the cycle is shown in Fig. 7. The points (a) and (c) indicate the MP and MPD conditions, respectively. The condition of maximum efficiency occurs at point (b). Therefore, the arc (a)–(b) shows the optimum working conditions when power output is the main interest. On the normalized MPD curve, the arc (c)–(b) represents the optimum working conditions if the engine size and economic aspects are the main interests. The point (c') has the advantage of having η correspond to $\eta_{\dot{w}_{d\max}}$. The point (a') refers to $\eta_{\dot{w}_{\max}}$ for the engine design in the MPD analysis. The point (e) is the intersection of the curves. When the working conditions at point (e) are determined, the engine has the following lower and upper power bounds:

$$1 > (\dot{W}_{\text{net}}/\dot{W}_{\max})_e > (\dot{W}_{\text{net}}/\dot{W}_{\max})_{c'}, \quad (10)$$

$$1 > (\dot{W}_d/\dot{W}_{d\max})_e > (\dot{W}_d/\dot{W}_{d\max})_{a'}. \quad (11)$$

These two inequalities order the efficiencies as follows:

$$\eta_{\dot{w}_{d\max}} > \eta_e > \eta_{\dot{w}_{\max}}. \quad (12)$$

The arc (e)–(c') constitutes the optimum working region. The points (e) and (c') show good agreement with the results of a recent study by De Vos [10] who stated that the optimal point for applications lies somewhere between the MP and $\eta_{\max}$ points. Point (e) is important for high-power designs and point (c') for engine size.

CONCLUSION

In the present study, we used polytropic processes in the power and displacement pistons to create a finite temperature difference at the regenerator for the purpose of obtaining a realistic RHX. For the

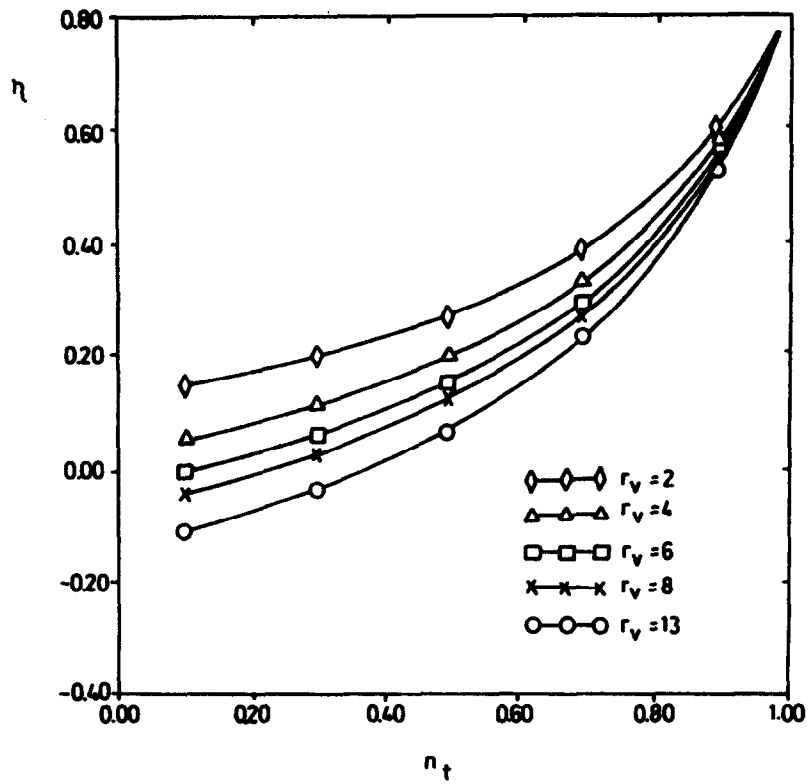


Fig. 6. The thermal efficiency η vs n_t for different cycle—volume ratios r_v ($\psi = 0.2$, $k = 1.4$).

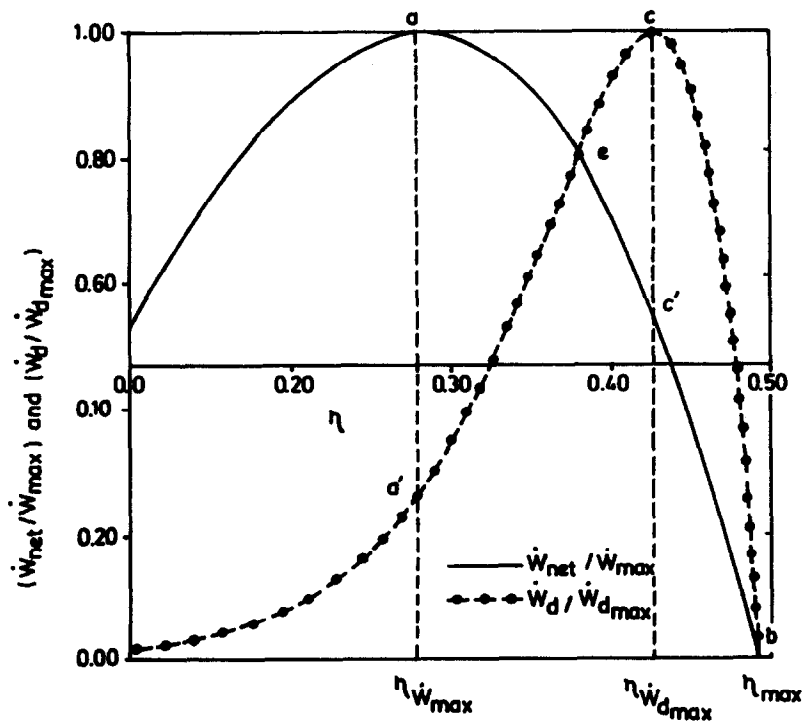


Fig. 7. A detailed view of the optimal cycle region ($\psi = 0.2$, $n_t = 0.75$).

determination of design parameters to reduce the engine size, the MPD technique was applied and the thermal design bounds of a Stirling heat engine determined.

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NOMENCLATURE

c_v = Constant—volume specific heat	T_2 = Minimum cycle temperature at the displacement-piston exit
k = Ratio of specific heats c_p/c_v	$\dot{W}_d$ = Cycle power density $\dot{W}_{net}/v_4$
$\dot{m}$ = Mass-flow rate	$\dot{W}_{d,max}$ = Maximum cycle—power density
n_c = Polytropic exponent at the displacement piston	$\dot{W}_{r,max}$ = Maximum cycle—power output
n_t = Polytropic exponent at the power piston	$\dot{W}_{net}$ = Net cycle—power output
$\dot{Q}_H$ = Rate of polytropic heat transfer from the high-temperature heat source to the working fluid	v = Specific volume
$\dot{Q}_L$ = Rate of polytropic heat transfer from the working fluid to the low-temperature heat source	v_4 = Maximum specific volume
R = Universal gas constant	v_2 = Minimum specific volume
r_v = Compression ratio v_2/v_4	ψ = Temperature ratio T_2/T_4
T_4 = Maximum cycle temperature at the power-piston exit	η = Thermal efficiency of the cycle
	η_{max} = Maximum thermal efficiency
	$\eta_{wd,max}$ = Cycle thermal efficiency at maximum cycle—power density
	$\eta_{w,max}$ = Cycle thermal efficiency at maximum cycle—power output